

## AUDITS OF PAPER RECORDS TO VERIFY ELECTRONIC VOTING MACHINE TABULATED RESULTS

*By Dr. Elizabeth Clarkson\**

In December 2012, I requested access to voting-machine records to conduct an audit and ascertain the accuracy of the reported machine-generated counts compared to the machine records recorded at the time a vote was cast. The Sedgwick County Election Office had never verified the accuracy of machine-vote counts since the machines were purchased in 2006. My request was refused.

In June 2013 I filed a lawsuit requesting access to voting machine records in my voting precinct of Sedgwick County, Kansas. This request was denied because the voting-machine records I requested are considered ballots and are exempt from the Kansas Open Records Law.

In November 2014, I filed a request for a recount of the general election and wanted to use those voting-machine records to verify the count. This was refused. The Sedgwick County Election Office informed me that using those records to conduct a recount was not allowed.

In February 2015, I filed a lawsuit requesting access to voting-machine records. In February 2016, a motion by the plaintiff to deny me access to those records was successful. In short, even though voters in Sedgwick County, Kansas, use voting machines with a paper trail, that paper trail is not public and no one is allowed to look at those records, even at their own expense. Not for a recount. Not to verify the accuracy of the reported results.

As a result of the publicity surrounding my lawsuit to obtain voting-machine records, many people have expressed interest in my work analyzing voting results. I would like to take this opportunity explain why this type of audit is necessary, to describe in detail the audit I would like to conduct, and finally to give recommendations regarding recently proposed legislation that would require such audits for certifying the official votes after an election.

The records I am seeking are rolls of paper, similar to a cash register paper tape. They are called Real Time Audit Logs (R.T.A.L.).

Real Time Audit Logs are created in real time – as the voters are making their choices. This physical paper trail is a minimum requirement for assessing the accuracy of the machine-reported results. Without it, we are not conducting verification worthy of being called an “audit”.

Computer files and electronic equipment cannot be made secure enough for this purpose<sup>1</sup>. Using electronic ballot equivalents or any other type of

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1. See PATRICK MCDANIEL ET AL., EVEREST: EVALUATION AND VALIDATION OF

electronically stored files breaks the transparent connection between the input and the output. We must verify our results with physical paper records that have been sealed and kept in the custody of election officials since the time the vote was cast. Anything less will not suffice.

Either the R.T.A.L. records are used to verify election results, or we cannot have confidence that our votes have been tabulated correctly on those machines.

### I. WHY THE AVAILABLE DATA SUPPORTS, BUT DOES NOT PROVE, THAT ELECTRONIC VOTING MACHINES ARE RIGGED.

For this analysis, I will be using only publicly available data. I am interested in testing whether or not vote rigging by machines might be occurring. Since that is the hypothesis of interest, the null hypothesis is the opposite. For the null, we will assume that no vote rigging is occurring, and check statistics to see if they are in line with that assumption.

It's important to distinguish between the null and alternative hypothesis as they relate to our question of electronic voting machine manipulation and the null and alternative statistical hypothesis tested. When we are looking at correlations without a direct connection (as R.T.A.L. records would provide), we cannot conclusively identify a cause for any suspicious pattern detected. When we reject the null, we can only conclude that the data is consistent with the voting machine manipulation hypothesis, not that such manipulation is occurring.

The statistical hypothesis being tested is the relationship between the number of votes cast in a precinct with the Republican share of the votes cast. Under the null hypothesis (no rigging of voting machines), the expectation is that there will be a negative relationship between the two variables for small precincts, but that large precincts instead show stability with respect to the share of the Republican vote.

If the alternative hypothesis (voting machines manipulated) is true and favors Republicans, then we can expect larger discrepancies between the true and reported votes leading to positive relationship between the number of votes cast and the Republican share of those votes. If the alternative hypothesis is true and favors Democrats, then we can expect larger discrepancies between the true and reported votes leading to a negative relationship between the number of votes cast and the Republican share of those votes.

The expectation under the null can be disputed. It's one reason that a result that rejecting the null hypothesis is not proof of voting machine manipulation. It can only be described as supportive of that hypothesis.

Using the cumulative summation percent model  $\mu = \frac{1}{m} \sum_{i=1}^m \left( \frac{r_i}{n_i} \right)$ , with  $m$  = number of precincts,  $r_i$  and  $n_i$  the number of republican votes and total votes cast in the  $i^{th}$  precinct.

A sequence of random variables can be created:

$$R_m = \sum_{i=0}^m \binom{m}{i} i - 1 \dots m \rightarrow R_m \sim n(\mu, \sigma)$$

It follows that:  $\epsilon_m = R_m - R_{m-1} \dots \sim n(0, \sigma/\sqrt{m})$ .<sup>2</sup>

We can examine this statistic for various races. A graphical display of this model is straightforward in interpretation. Under the assumptions of the null hypothesis the statistic  $\bar{R}_m$  converge rapidly to  $\mu$  as  $m$  increases, specifically the random values (in this case, the percentage of Republican votes) are uncorrelated with the sequence of the summation. Under the null hypothesis, different sorts of voting equipment would all show the same random *pattern*, albeit different *means* might be expected, assuming that the choice of equipment has geographic – and hence demographic – correlations.

Here is a picture of Wisconsin results for Governor, 2012:

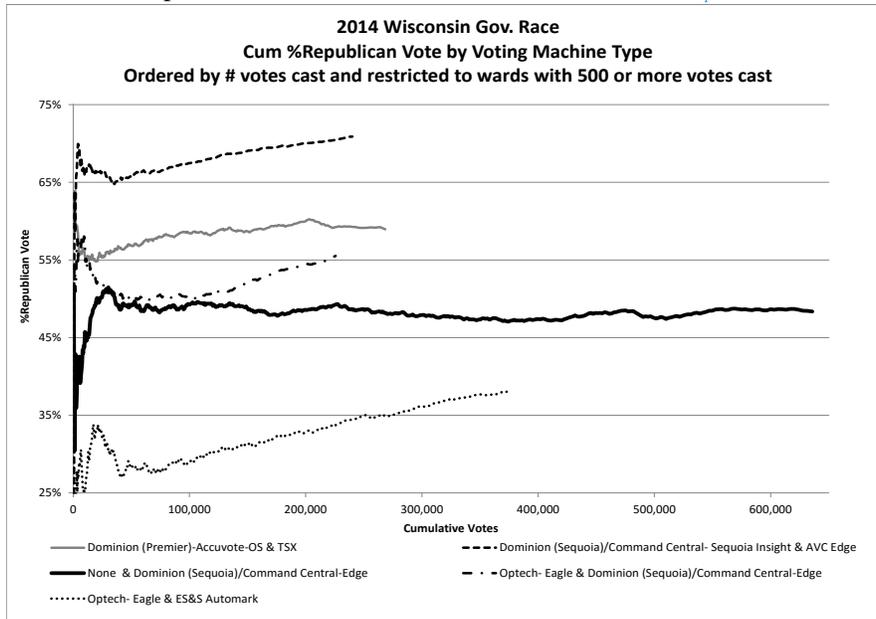


Figure 1: 2014 Wisconsin Governor Election Results

The heavy black line, which represents paper voting systems, shows a classic pattern of random variation. While the three dashed lines show clear positive relationships between the number of votes cast and the percent of the Republican vote for the three electronic voting systems. An ambiguous pattern is illustrated by the gray line. We reject the null ( $H_0$ ) and conclude the alternative ( $H_a$ ) for the datasets shown by the three dashed lines. We fail to reject the null for the solid black and gray lines. I have yet to hear of a more plausible explanation for the non-random patterns illustrated by the dashed lines in

Figure 11 than illegal and illicit manipulation of the electronic voting equipment.

However, this analysis only tells us that a suspicious pattern exists. The pattern shown by the dashed lines fits our assumption that voting machines are being rigged. But it NOT proof of manipulation because we don't actually know the cause, and there might exist another explanation for this pattern. For example, the choice of voting machinery might correlate with some other population demographic that hasn't yet been identified. Whatever you think of the probability of this explanation, it is important to realize that it is non-zero and therefore seek better evidence. Make allowances for the unknown, particularly when making judgments about a field outside your expertise. There is also a possibility that there exists some factor you are completely oblivious of.

I can explore other factors analytically. Yes, there are some correlates, such as percent of registered Republicans, that explain part of the anomalous pattern I see. It is not sufficient to quell my fears.<sup>3</sup>

One can search endlessly for other explanations. Even if found, their existence would not constitute proof that such factors were the cause of the pattern. I want to either determine that our votes on these machines are being tabulated correctly, or have convincing evidence that the current system cannot be trusted. There is no reason to have confidence in election results that were counted without transparency. In fact, we should not.

If we fail to reject the null after an audit, we can only say that if the voting machine were rigged, we could not detect it. This, by the way, is what qualifies as a gold star in audit results: We can't find anything wrong and we looked really hard. Is it any wonder nobody is interested in having them done?

If after an audit I can conclude that the electronic voting machine results were counted accurately, it will be great relief to me and other Kansans. I could conclude that if election manipulation is occurring, it's not occurring during the final tabulation of votes at the end of the day.

Finally, even if I conclude that our election results were rigged, it does not indicate anything about who has engineered the discrepancies. It only establishes that they occurred.

## II. KANSAS VOTING RESULTS

The 2014 Kansas Senate race resulted in the largest deviation from the pre-election polling results in the nation.<sup>4</sup> It is also the only 2014 Kansas race that the Secretary of State's office has released precinct level results.<sup>5</sup> So my analysis

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3. Elizabeth Clarkson, *How Trustworthy are Electronic Voting Systems in the US?*, ROYAL STAT. SOC'Y: SIGNIFICANCE (June 5, 2015), <https://www.statslife.org.uk/significance/politics/2288-how-trustworthy-are-electronic-voting-systems-in-the-us>.

4. Kathy Dopp, *Were 2014 United States Senatorial and Gubernatorial Elections Manipulated?*, ELECTION MATHEMATICS (Apr. 17, 2015, 10:18PM), <http://kathydopp.com/wordpress/?p=408>.

5. *2014 Kansas Primary Election Results by Precinct*, KAN. SEC'Y OF ST., [http://www.sos.ks.gov/elections/14elec/2014\\_general\\_us\\_senate\\_official\\_results.xls](http://www.sos.ks.gov/elections/14elec/2014_general_us_senate_official_results.xls) (last visited Mar. 4, 2016).

has focused on this race.

Looking at just the western half of Kansas, I separated out the counties that use electronic voting machines from those that use paper ballots. Figure 2 shows western Kansas not only shows a solid support for Pat Roberts, winning approximately 70% of the vote, but also demonstrates the expected pattern. Initially, it's a random pattern with large deviations, but the size of the deviations slow down quickly. The slow decrease to the average as the precinct size increase can be explained by the fact that, even in western Kansas, city dwellers are not as heavily Republican as in relatively more rural areas. There is little difference in the results between the two systems.

In short, these low population density precincts in western Kansas show no signs of the non-random pattern that indicates election rigging.

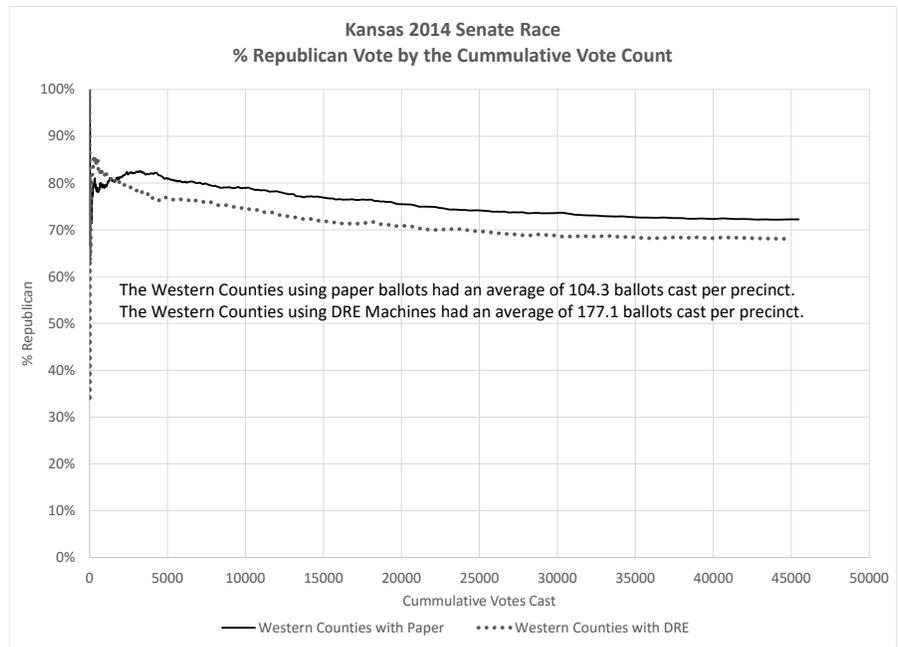


Figure 2: Western Counties with and without DRE

Figure 3 adds Johnson and Douglas counties to Figure 2. Both of those urban areas show:

- (1) Considerably lower support for the Republican candidate than western KS. This is expected because urban areas have a larger percentage of registered Democrats.
- (2) An increase in support for the Republican candidate as precinct size grows larger starting at about 300 votes cast per precinct for Johnson County and 500 votes cast per precinct for Douglas County. This increase in support for the Republican candidate as the number of votes cast increases is the anomalous pattern that concerns me.

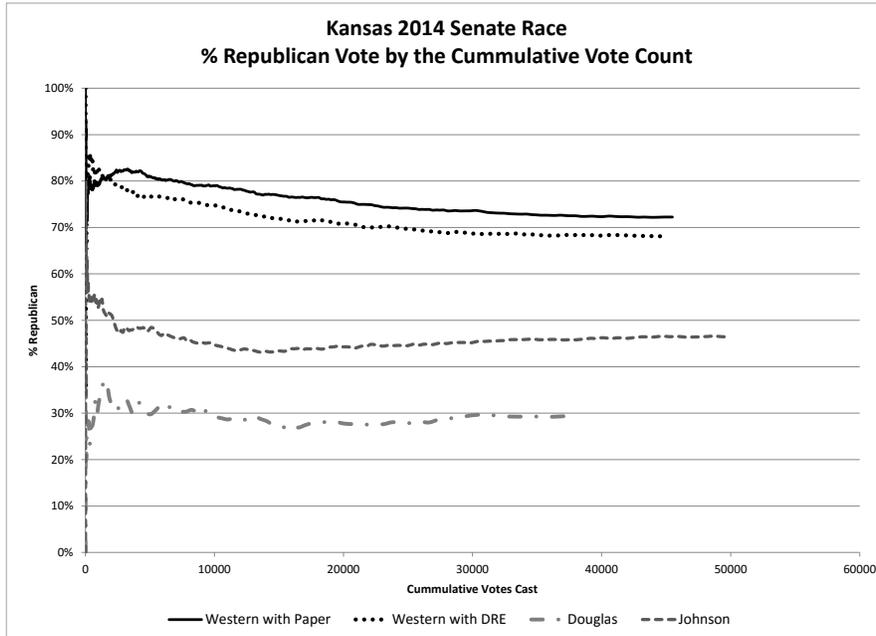


Figure 3: Western Counties Plus Douglas and Johnson

Location	Type	Total Ballots Cast	Ballots per Precinct
Western Ks	Paper	45483	104.3
Combined Counties	Machine	44800	177.1
Douglas County	Paper	37758	406.0
Johnson County	Machine	192677	431.0

Table 1: Kansas chart voting details

Figure 4 focus in on Sedgwick County and its surrounding area. While the overall support for the Republican candidates is slightly lower than in western Kansas, it clearly shows the same pattern as the western Kansas counties. Wichita, on the other hand, shows the unexplained pattern of an increase in the Republican vote share correlating with the number of votes cast in a precinct.

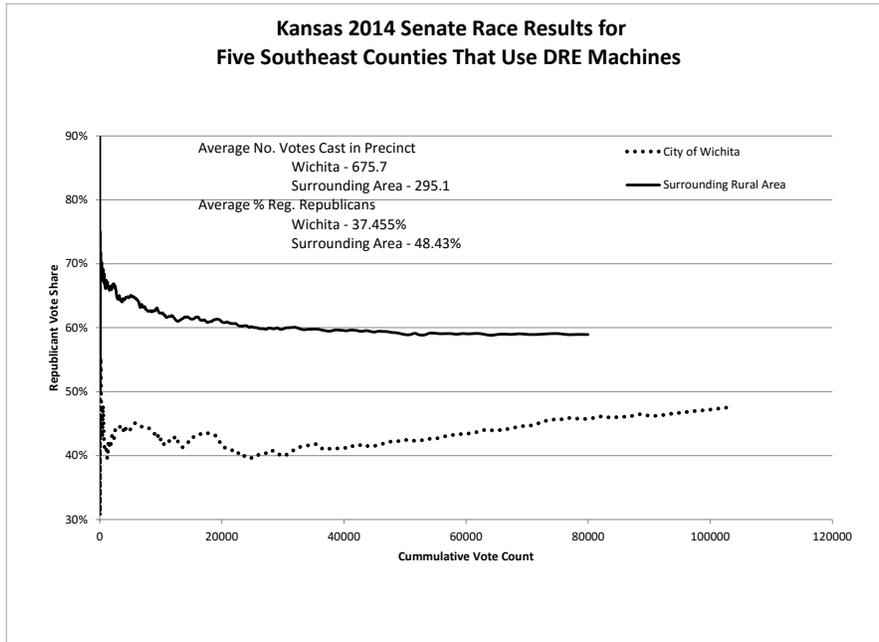


Figure 4: Wichita & Surrounding Area

Figure 5 shows the 2014 Wichita data<sup>6</sup> for the Governor’s race, the Secretary of State race, and the Attorney General race. The similarity of the pattern is not entirely unexpected, but clearly whatever is causing this anomaly is affecting all the Republican candidates. The solid black line across the bottom is the same analysis applied to the registered voters in Sedgwick County.

6. November 4th, 2014 General Election Official Results – Sedgwick County, SEDGWICK CTY. (Nov. 13, 2014, 11:40:17 PM), [http://www.sedgwickcounty.org/elections/election\\_results/Gen14/index.html](http://www.sedgwickcounty.org/elections/election_results/Gen14/index.html).

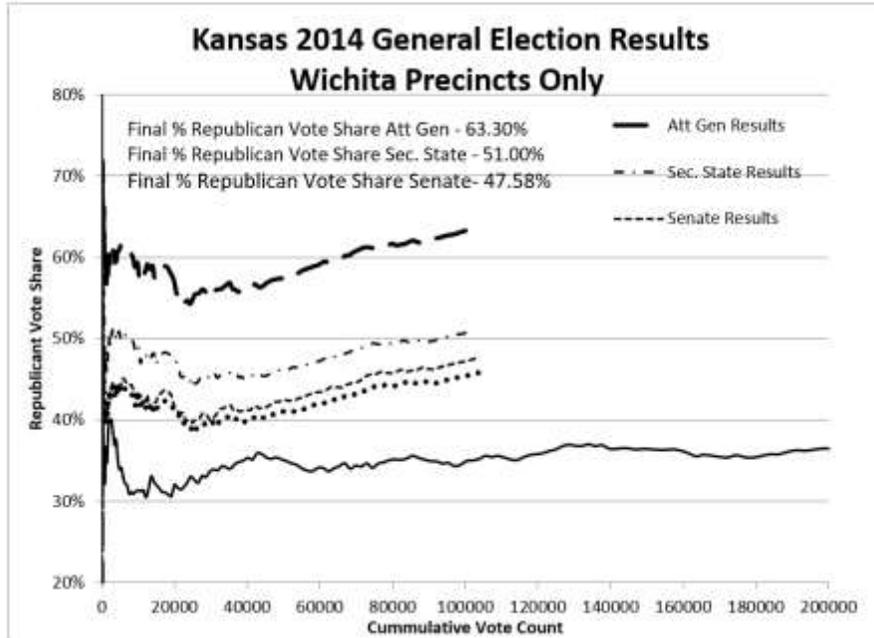


Figure 5: Wichita 2014 Results

### III. THE MECHANICS OF AUDITING ELECTION RESULTS USING R.T.A.L. RECORDS.

One reason that Ms. Tabitha Lehman, the Sedgwick County Election Commissioner, contends that she should not be compelled to produce the records I have requested is that it would be a burdensome task for her office. On the one hand, the difficulty of the task is a valid point. It's a boring, toilsome job to put those records in order and provide either supervised access or public viewing. On the other hand, given that I expect to be charged for the time spent on providing those records it's rather hard to credit it as legitimate reason not to grant my request.

Ms. Lehman makes a valid point that running an audit or other verification using paper records will be burdensome. It is a difficult, time-consuming and detail-focused task. The following is a detailed plan for accomplishing it.

First, we must select one or more polling places. I've asked for a specific one - Countryside Christian Church, but I am fine with a random sample of several polling stations throughout Sedgwick County.

For this analysis, I've decided to limit it to the R.T.A.L.s for election-day results. I can forgo the mail-in ballots, the early voting, and the provisional and paper ballots cast at polling places. This does not mean that those are fine or I don't care about them. It's because the difficulty of including those results comes at a high resource cost while the gain in likelihood of detecting fraud over using the limited sample is small. That's why I think the best way to accomplish

an audit with limited resources is to zero in on the results reported for in-person machine results from Nov. 4, 2014.

Once a polling place (or five) have been selected, a list of the serial numbers of the voting machines for each polling place is generated. Our sample will consist of all R.T.A.L. records for all machines located in the selected polling stations on Election Day.

According to Ms. Lehman, there are approximately four dozen boxes of R.T.A.L. records from the 2014 general election. Every single roll of R.T.A.L. paper tapes in those boxes will have to be individually examined; the serial number on the roll compared to the sample list. In short, before we can even start the process, there are hours and hours of tedious and exacting clerical work just to organize the files and separate out the records that comprise our sample.

Once we have separated out the physical records of our sample, we begin the process of examining each ballot. The trouble and expense required can vary tremendously depending on the requirements the court decides are appropriate. These may involve the presence of other individuals, perhaps representative of all parties as well as restrictions such as allowing me to examine the records, but not touch them. Will I be allowed to photograph them? Will they be willing to make copies while I, or someone else, observe? Who else will be permitted to observe the records? I have set up the Show Me The Votes Foundation – to handle fundraising for this part of the project, but I still have no idea how much they will charge to do this. The elections office isn't giving out any estimates.

Finally, once the in-person physical observations have been made and individual ballot results recorded in my database, I can begin the statistical analysis.

#### IV. THE STATISTICAL ANALYSIS AND INTERPRETATION OF AUDIT RESULTS.

In setting up null and alternative hypothesis, the null should be the hypothesis that the experiment is designed to reject and the alternative is the hypothesis the experiment is designed to prove. Thus, when the null is rejected, the alternative will be considered proven.

- Each race in a single precinct constitutes an experimental unit.
  - $N$  = the total number of units in the entire sample.
- The statistic for each unit will be the error rate – the difference of each computer reported result for the Republican candidate compared with manual human count performed on the R.T.A.L. records.
- The mean error rate is the test statistic.
  - $\bar{e} = \frac{1}{N} \sum_{i=1}^N e_i \quad i = 1 \dots N$ 
    - $\bar{e} \sim n(0, \sigma^2 / (N - 1))$
  - Under the null hypothesis, this error rate is expected to be distributed as a normal random variable with a mean of zero. The

expected standard deviation will be computed the from the sample values.

- If the average deviates significantly from zero, then those differences constitute proof of voting machine tampering.
- A one-sided test would be appropriate as all indications are that suspected manipulations in Kansas benefit the Republican Party Establishment.
- Two-sided results indicate the probability without assuming benefits go the Republican Party Establishment.
- Both two sided and one-sided results will be provided.
- The following hypotheses tests will be used:

For a two sided hypothesis test,

$$H_0: \bar{e} = 0$$

$$H_a: \bar{e} \neq 0$$

For a one sided hypothesis test,

$$H_0: \bar{e} \geq 0$$

$$H_a: \bar{e} < 0$$

I hope to someday get access to the needed records and publicly release the results of such a test.

## V. THE 2016 IOWA DEMOCRATIC CAUCUS CORRECTIONS

I will illustrate the method with the recent corrections to the Iowa Democratic Caucus results. It's a convenient dataset recently and publicly available.

There were five precincts out of 1681 total that had corrections.<sup>7</sup> That's an overall error rate slightly below 0.03%. There were two measures of the error given, the change in Delegate Counts, which are discrete and a corresponding decimal value called Delegate Equivalents. The changes made are shown in Table 2.

County	Precinct	Change in Delegate Count			Change in Delegate Equivalents		
		HRC	BS	MO	HRC	BS	MO
Marion	Knoxville 3	-1	1	0	-0.1300	0.1300	0
Woodbury	Oto	-1	1	0	-0.1500	0.1500	0
Osceola	Ashton	0	-1	1	0	-0.0167	0.0167
Story	Sherman	1	-1	0	0.2300	-0.2300	0
Poweshiek	1st Ward (Grinnell)	-1	1	0	-0.0720	0.0720	0
<b>Totals</b>		<b>-2</b>	<b>1</b>	<b>1</b>	<b>-0.1220</b>	<b>0.1053</b>	<b>0.0167</b>
<b>Two-sided Probability Under the Null</b>		<b>.3125</b>	<b>.1875</b>	<b>.5</b>	<b>.3006</b>	<b>.2229</b>	<b>NA</b>

Table 2: Ohio 2016 Caucus Corrections

7. Jennifer Jacobs, *Iowa Dems Fix Errors in Caucus Results, Say Clinton Still Winner*, THE DES MOINES REG. (Feb. 7, 2016, 6:27 PM), <http://www.desmoinesregister.com/story/news/elections/presidential/caucus/2016/02/07/iowa-dems-fix-errors-caucus-results-say-clinton-still-winner/79967552/>.

We can compute the mean and standard deviation of the decimal value and make a determination regarding whether the mean value of the changes is statistically significantly different from zero. If it is, that indicates bias. Bias is consistent with the hypothesis of election rigging.

Looking at the scores of each candidate in term (note, these are not independent variables), nothing seems particularly out of kilter. The two-sided probabilities given for these results tell us that they are not particularly unusual under the null hypothesis. When these probabilities fall below 0.05, further investigation is worthwhile. For these results, no further investigation is needed. We fail to reject the null and can conclude that there is no evidence of bias.

## VI. COMPUTATIONS FOR THE PROBABILITIES GIVEN IN TABLE 2

### A. *Delegate Counts*

The null hypothesis for binomial calculations sets  $H_0: p = 0.5$ , that is any change made to a candidates score is equally likely to be positive or negative. Choosing the two-sided alternative,  $H_a: p \neq 0.5$ .

Hillary Clinton has a net loss of 2 delegates out of 4 changes to her results. The probability was computed as a binomial distribution with  $n = 4$  and  $p = .5$ .  $P(X \leq 1) = .3125$ .

Bernie Sanders has a net gain of one delegate out of 5 changes to his results. The probability was computed as a binomial distribution with  $n = 5$  and  $p = .5$ .  $P(X \leq 1) = .1875$ .

Martin O'Malley only had one correction and it was in his favor, so with  $n = 1$  and  $p = .5$ ,  $P(X=0) = .5$

### B. *State Delegate Equivalent (SDE)*

The null hypothesis for delegate equivalents is that the mean size of the corrections in SDE for each candidate is not distinguishable from zero. I also assume the corrections for each candidate will follow a normal distribution.

The Null Hypothesis:  $H_0: \mu = 0$ . This reflects the assumption that corrections made to a candidates score is equally likely to be positive or negative.

The Two-sided Alternative,  $H_a: \mu \neq 0$ . This reflects that we are testing for both unusually high or unusually low mean values.

Clinton has a mean loss of 0.0305 SDE out of 4 changes to her results. The standard deviation for those changes was 0.1768. Assuming corrections are distributed normally with a mean of zero and a standard deviation of 0.1768,  $P(|x| < .0305) = 0.3006$ .

Sanders has a mean gain of 0.0211 SDE out of 5 changes to his results. The standard deviation for those changes was 0.1546. Assuming corrections are distributed normally with a mean of zero and a standard deviation of 0.1546,  $P(|x| < .0211) = 0.2229$ .

Martin O'Malley had a mean gain of 0.0167. We have no way to compute a standard deviation for a sample size of 1. No probability for the S.D.E. corrections was computed for this candidate.

## VII. CONCLUSION

Since we fail to reject the null for all candidates, we can conclude there was no bias or systematic effect favoring any candidate in the errors that were reported and corrected in the Iowa Caucus results.

The available evidence points to election fraud via electronic equipment. Caucus votes are not as easily manipulated, as demonstrated by the Iowa Caucuses showing no sign of having been rigged.